# ••• Master List of Formulas

## **CHAPTER 1 INTRODUCTION AND DESCRIPTIVE STATISTICS**

NONE.

# CHAPTER 2 FREQUENCY DISTRIBUTIONS IN TABLES AND GRAPHS

 $\Sigma x$  (Frequency)

 $\frac{\Sigma x}{n}$  (Relative frequency)  $\frac{\Sigma x}{n} \times 100$  (Relative percent)

# **CHAPTER 3 SUMMARIZING DATA: CENTER TENDENCY**

 $\mu = \frac{\Sigma x}{N}$  (Population mean)  $M = \frac{\Sigma x}{n}$  (Sample mean)  $M_w = \frac{\Sigma (M \times n)}{\Sigma n}$  (Weighted sample mean)

# CHAPTER 4 SUMMARIZING DATA: VARIABILITY

R = L - S (Range)

 $IQR = Q_3 - Q_1$  (Interquartile range)

 $SIQR = \frac{IQR}{2}$  (Semi-interquartile range)

 $SS = \Sigma(x - \mu)^2$  (Definitional formula for the sum of squares in a population)

 $SS = \Sigma x^2 - \Sigma \frac{(\Sigma x)^2}{N}$  (Computational formula for the sum of squares in a population)

 $\sigma^{2} = \frac{SS}{N} \text{ [Population variance]}$   $\sigma = \sqrt{\sigma^{2}} = \sqrt{\frac{SS}{N}} \text{ [Population standard deviation]}$   $SS = \Sigma(x - M)^{2} \text{ [Definitional formula for sum of squares in a sample]}$   $SS = \Sigma x^{2} - \frac{(\Sigma x)^{2}}{n} \text{ [Computational formula for sum of squares in a sample]}$  df = n - 1 [Degrees of freedom for sample variance]  $s^{2} = \frac{SS}{n-1} = \frac{SS}{df} \text{ [Sample variance]}$   $SD = \sqrt{s^{2}} = \sqrt{\frac{SS}{n-1}} = \sqrt{\frac{SS}{df}} \text{ [Sample standard deviation]}$ 

#### **CHAPTER 5 PROBABILITY**

- $p(x) = \frac{f(x)}{sample \ space} \text{ [Simple probability]}$  $p(U / P) = \frac{p(P / U)p(U)}{p(P)} \text{ [Bayes's theorem]}$
- $\mu = \Sigma(xp)$  (Mean of a probability distribution)

 $\sigma^2 = \Sigma((x - \mu)^2 p)$  (Variance of a probability distribution)

 $\sigma = \sqrt{\sigma^2} = \sqrt{\Sigma((x - \mu)^2 p)}$  (Standard deviation of a probability distribution)

 $\sigma = \sqrt{\sigma^2} = \sqrt{(\Sigma(x^2p) - \mu^2)}$  (Computing formula for variance of a probability distribution)

 $\sigma = \sqrt{(\Sigma(x^2 \rho) - \mu^2)}$  (Computing formula for standard deviation of a probability distribution)

 $\mu = np$  (Mean of a binomial probability distribution)

 $\sigma^2 = np(1-p) = npq$  (Variance of a binomial probability distribution)

 $\sigma = \sqrt{np(1-p)} = \sqrt{npq}$  (Standard deviation of a binomial probability distribution)

# CHAPTER 6 PROBABILITY, NORMAL DISTRIBUTIONS, AND Z SCORES

 $z = \frac{x - \mu}{z}$  (z transformation for a population of scores)

 $z = \frac{x - M}{SD}$  (z transformation for a sample of scores)

## CHAPTER 7 PROBABILITY AND SAMPLING DISTRIBUTIONS

 $z = \frac{x - \mu}{z}$  (z transformation for a population of scores)

 $z = \frac{x - M}{SD}$  (z transformation for a sample of scores)

 $z = \frac{M - \mu}{\sigma_M}$  (z transformation for a distribution of sample means)

 $\sigma_M = \sqrt{\frac{\sigma^2}{n}} = \frac{\sigma}{\sqrt{n}}$  (Standard error of the mean)

# CHAPTER 8 HYPOTHESIS TESTING: SIGNIFICANCE, EFFECT SIZE, AND POWER

 $z_{obt} = \frac{M-\mu}{\sigma_M}$  (Test statistic for the one-sample z test)

 $d = \frac{M-\mu}{\sigma}$  (Cohen's *d* effect size measure for the one-sample *z* test)

# CHAPTER 9 TESTING MEANS: ONE-SAMPLE AND TWO-INDEPENDENT-SAMPLE *t* TESTS

## One-Sample t

$$t_{obt} = \frac{M - \mu}{s_M}$$
 (Test statistic for the one-sample *t* test)  
 $s_M = \sqrt{\frac{s^2}{n}}$  (Estimated standard error)

df = n-1 (Degrees of freedom for the one-sample t test)

#### Two-Independent-Sample t

$$t_{obt} = \frac{(M_1 - M_2) - (\mu_1 - \mu_2)}{s_{M_1 - M_2}} \text{ (Test statistic for the two-independent-sample t test)}$$

$$s_{M_1 - M_2} = \sqrt{\frac{s_p^2}{n_1} + \frac{s_p^2}{n_2}} \text{ (Estimated standard error for the difference)}$$

$$s_p^2 = \frac{s_1^2 (df_1) + s_2^2 (df_2)}{df_1 + df_2} \text{ (Pooled sample variance for unequal sample sizes)}$$

 $df = (n_1 - 1) + (n_2 - 1)$  (Degrees of freedom for the two-independent-sample t test)

#### **Effect Size**

$$d = \frac{M - \mu}{SD}$$
 (Estimated Cohen's *d* for the one-sample *t* test)

 $\frac{M_1 - M_2}{\sqrt{s_p^2}}$  [Estimated Cohen's *d* for the two-independent-sample *t* test]

 $\eta^2 = \frac{t^2}{t^2 + dt}$  [Eta-squared estimate of proportion of variance; used for all t tests]

 $ω^2 = \frac{t^2 - 1}{t^2 + dt}$  (Omega-squared estimate of proportion of variance; used for all *t* tests)

# CHAPTER 10 TESTING MEANS: THE RELATED-SAMPLES *t* TEST

#### **Related-Samples** t

 $t_{obt} = \frac{M_D - \mu_D}{s_{MD}}$  (Test statistic for the related-samples *t* test)  $s_{MD} = \sqrt{\frac{s_D^2}{n_D}} = \frac{s_D}{\sqrt{n_D}}$  (Estimated standard error for difference scores)

 $df = n_D - 1$  (Degrees of freedom for related-samples t test)

#### **Effect Size**

 $d = \frac{M_D}{s_D}$  [Estimated Cohen's *d* for related-samples *t* test]  $\eta^2 = \frac{t^2}{t^2 + df}$  [Eta-squared estimate of proportion of variance; used for all *t* tests]

 $\omega^2 = \frac{t^2 - 1}{t^2 + dt}$  (Omega-squared estimate of proportion of variance; used for all t tests)

## CHAPTER 11 ESTIMATION AND CONFIDENCE INTERVALS

 $M \pm z(\sigma_M)$  (The estimation formula for a one-sample z test)

 $M \pm t(s_M)$  (The estimation formula for a one-sample t test)

 $M_1 - M_2 \pm t(s_{M_1 - M_2})$  (The estimation formula for a two-independent-sample t test)

 $M_D \pm t(s_{MD})$  (The estimation formula for a related-samples t test)

## CHAPTER 12 ANALYSIS OF VARIANCE: ONE-WAY BETWEEN-SUBJECTS DESIGN

#### **Table for One-Way Between-Subjects ANOVA**

Source of Variation	SS	df	MS	<b>F</b> <sub>obt</sub>
Between groups		<i>k</i> – 1	$\frac{SS_{BG}}{df_{BG}}$	$rac{MS_{BG}}{MS_{E}}$
Within groups (error)		N-k	$\frac{SS_E}{df_E}$	
Total		N – 1		

#### **Between-Subjects Design**

- $F_{obt} = \frac{MS_{BG}}{MS_E}$  (Test statistic for the one-way between-subjects ANOVA)
- $MS = \frac{SS}{dt}$  (Mean square for each source of variation; used for all ANOVA tests)
- $df_{BG} = k 1$  (Degrees of freedom between groups)
- $df_E = N k$  (Degrees of freedom error)
- $df_T = N 1$  (Degrees of freedom total)

#### Effect Size (Between-Subjects Design)

- $R^2 = \eta^2 = \frac{SS_{BG}}{SS_T}$  [Eta-squared estimate for proportion of variance]
- $\omega^{2} = \frac{SS_{BG} df_{BG}[MS_{E}]}{SS_{T} + MS_{E}}$  (Omega-squared estimate for proportion of variance)

#### **Post Hoc Tests**

 $t_{\alpha}\sqrt{MS_{E}\left(\frac{1}{n_{1}}+\frac{1}{n_{2}}
ight)}$  (Fisher's LSD formula)  $q_{\alpha}\sqrt{\frac{MS_{E}}{n}}$  (Tukey's HSD formula)

# CHAPTER 13 ANALYSIS OF VARIANCE: ONE-WAY WITHIN-SUBJECTS (REPEATED-MEASURES) DESIGN

## Table for One-Way Within-Subjects (Repeated-Measures) ANOVA

Source of Variation	SS	df	MS	<b>F</b> <sub>obt</sub>
Between groups		<i>k</i> – 1	$\frac{SS_{BG}}{df_{BG}}$	MS <sub>BG</sub> MS <sub>E</sub>
Between persons		<i>n</i> – 1	$\frac{SS_{BP}}{df_{BP}}$	
Within groups (error)		(k-1)(n-1)	$\frac{SS_E}{df_E}$	
Total		( <i>kn</i> – 1)		

#### Within-Subjects Design

 $F_{obt} = \frac{MS_{BG}}{MS_E}$  (Test statistic for the one-way within-subjects ANOVA)

 $df_{BG} = k - 1$  (Degrees of freedom between groups)

 $df_{BP} = n - 1$  (Degrees of freedom between persons)

 $df_E = (k-1)(n-1)$  (Degrees of freedom error)

 $df_T = (kn) - 1$  (Degrees of freedom total)

#### Effect Size (Within-Subjects Design)

 $\eta_{P}^{2} = \frac{SS_{BG}}{SS_{T} - SS_{BP}} \text{ (Partial eta-squared)}$ 

 $\omega_{p}^{2} = \frac{SS_{BG} - df_{BG}(MS_{E})}{(SS_{T} - SS_{BP}) + MS_{E}} \text{ [Partial omega-squared]}$ 

# CHAPTER 14 ANALYSIS OF VARIANCE: TWO-WAY BETWEEN-SUBJECTS FACTORIAL DESIGN

## ANOVA Table for Two-Way Between-Subjects Factorial Design

Source of Variation	SS	df	MS	F
Factor A		р — 1	$\frac{SS_A}{df_A}$	$F_{A} = \frac{MS_{A}}{MS_{E}}$
Factor B		<i>q</i> – 1	$\frac{SS_B}{df_B}$	$F_B = \frac{MS_B}{MS_E}$
A×B		(p-1)(q-1)	$\frac{SS_{A \times B}}{df_{A \times B}}$	$F_{A\times B} = \frac{MS_{A\times B}}{MS_E}$
Error (within groups)		pq(n – 1)	$\frac{SS_E}{df_E}$	
Total		npq — 1		

#### **Between-Subjects Design**

- $F_A = \frac{MS_A}{MS_E}$  (Test statistic for the main effect of Factor A)
- $F_B = \frac{MS_B}{MS_E}$  (Test statistic for the main effect of Factor B)
- $F_{A\times B} = \frac{MS_{A\times B}}{MS_F}$  (Test statistic for the A × B interaction)
- $df_A = p 1$  (Degrees of freedom for Factor A)
- $df_B = q 1$  (Degrees of freedom for Factor B)
- $df_{A \times B} = (p 1)(q 1)$  (Degrees of freedom for the A × B interaction)
- $df_F = pq(n-1)$  (Degrees of freedom error)
- $df_T = npq 1$  (Degrees of freedom total)

#### Effect Size (Two-Way Between-Subjects ANOVA)

 $\eta_A^2 = \frac{SS_A}{SS_T}, \ \eta_B^2 = \frac{SS_B}{SS_T}$  [Eta-squared for main effects]  $\eta^2_{A\times B} = \frac{SS_{A\times B}}{SS_{\tau}}$  (Eta-squared for the interaction)  $\omega_A^2 = \frac{SS_A - df_A(MS_E)}{SS_T + MS_E}, \ \omega_B^2 = \frac{SS_B - df_B(MS_E)}{SS_T + MS_E} \text{ (Omega-squared for main effects)}$  $\omega_{A\times B}^{2} = \frac{SS_{A\times B} - df_{A\times B}(MS_{E})}{SS_{T} + MS_{T}}$  (Omega-squared for the interaction)

# **CHAPTER 15 CORRELATION**

#### **Correlation Coefficients**

 $r = \frac{SS_{XY}}{\sqrt{SS_{X}SS_{Y}}}$  (Pearson correlation coefficient)  $r_s = 1 - \frac{6\Sigma D^2}{n(n^2 - 1)}$  (Spearman rank-order correlation coefficient)  $r_{pb} = (\frac{M_{\gamma_1} - M_{\gamma_2}}{S_{\gamma_1}})(\sqrt{pq})$  (Point-biserial correlation coefficient)  $r_{\phi} = \frac{ad - bc}{\sqrt{ABCD}}$  (Phi correlation coefficient)

# Converting the Correlation Coefficient (r) to t and $\chi^2$

 $t^2 = \frac{r^2}{(1-r^2)/dt}$  (Formula for converting r to t)

 $\chi^2 = r_{\phi}^2 n$  (Formula for converting r to  $\chi^2$ )

#### **Effect Size**

 $r^2 = \eta^2$  (The coefficient of determination)

## **CHAPTER 16 LINEAR REGRESSION** AND MULTIPLE REGRESSION

#### **Method of Least Squares**

Y = bX + a (Linear equation for a straight line)

 $b = \frac{SS_{XY}}{SS_{Y}}$  (Slope of a straight line)

 $a = M_Y - bM_X$  (y-intercept for a straight line)

## **Analysis of Regression**

 $F_{obt} = \frac{MS_{\text{regression}}}{MS_{\text{residual}}}$  (Test statistic for analysis of regression and multiple regression)

 $df_{regression} = 1$  (Degrees of freedom regression with one predictor variable)

 $df_{\text{residual}} = n - 2$  (Degrees of freedom residual)

 $s_e = \sqrt{MS_{\text{residual}}}$  (Standard error of estimate)

# CHAPTER 17 NONPARAMETRIC TESTS: CHI-SQUARE TESTS

## **Chi-Square Tests**

 $\chi^2_{obt} = \Sigma \frac{[f_o - f_e]^2}{f_e}$  (Test statistic for the chi-square goodness-of-fit test and the chi-square test for independence)

df = k - 1 (Degrees of freedom for the chi-square goodness-of-fit test)

 $df = (k_1 - 1)(k_2 - 1)$  (Degrees of freedom for the chi-square test for independence)

## Effect Size (Chi-Square Test for Independence)

 $\phi^{2} = \frac{\chi^{2}}{n}$  (Effect size using the proportion of variance)  $\phi = \sqrt{\frac{\chi^{2}}{n}}$  (Effect size using the phi coefficient)  $V = \sqrt{\frac{\chi^{2}}{n \times df_{smaller}}}$  (Effect size using Cramer's V)

# CHAPTER 18 NONPARAMETRIC TESTS: TESTS FOR ORDINAL DATA

#### The Sign Test

 $z = \frac{x - np}{\sqrt{np(1-p)}}$  (Test statistic for the normal approximation of the sign test)

#### Wilcoxon Signed-Ranks T Test

 $z = \frac{T - \mu_T}{\sigma_T} \text{ [Test statistic for the normal approximation of the Wilcoxon 7]}$  $\mu_T = \frac{n(n+1)}{4} \text{ [The mean for the test statistic 7]}$  $\sigma_T = \sqrt{\frac{n(n+1)(2n+1)}{24}} \text{ [The standard deviation for the test statistic 7]}$ 

#### Mann–Whitney U Test

$$z = \frac{U - \mu_U}{\sigma_U}$$
 (Test statistic for the normal approximation of the Mann–Whitney U)

 $\mu_U = \frac{n_A n_B}{2}$  (The mean for the test statistic U)

 $\sigma_U = \sqrt{\frac{n_A n_B (n_A + n_B + 1)}{12}}$  (The standard deviation for the test statistic U)

## The Kruskal–Wallis H Test

 $H = \frac{12}{N(N+1)} \left( \Sigma \frac{R^2}{n} \right) - 3(N+1) \text{ (Test statistic for the Kruskal-Wallis } H \text{ test)}$ 

## **The Friedman Test**

 $\chi_R^2 = \frac{12}{nk(k+1)}\Sigma R^2 - 3n(k+1)$  (Test statistic for the Friedman test)