CHAPTER 5— ANSWERS TO EXERCISES

1.

a. The Z score for a person who watches more than 8 hours per day:

$$Z = \frac{8 - 2.94}{2.60} = 1.95$$

b. We first need to calculate the Z score for a person who watches 5 hours per day:

$$Z = \frac{5 - 2.94}{2.60} = 0.79$$

The area between Z and the mean is 0.2852. We then need to add 0.50 to 0.2852 to find the proportion of people who watch television less than 5 hours per day. Thus, we conclude that the proportion of people who watch television less than 5 hours per day is 0.7852. This corresponds to 786 people (785.99 = 0.7852 × 1,001).

c. 5.54 television hours per day corresponds to a Z score of +1.

$$Y = \overline{Y} + Z(S_Y) = 2.94 + 1(2.60) = 5.54$$

d. The Z score for a person who watches 1 hour of television per day is –.75. The area between the mean and Z is 0.2734.

$$\frac{1 - 2.94}{2.60} = -.75$$

The Z score for a person who watches 6 hours of television per day is 1.18. The area between the mean and Z is .3810.

$$\frac{6 - 2.94}{2.60} = 1.18$$

Therefore, the percentage of people who watch between 1 and 6 hours of television per day is 65.44% (0.2734 + 0.3810 = 0.6544 × 100).

2.

a. The 95th percentile corresponds to a Z score of about 1.65. Translating this into a raw score for the number of women needing shelter yields

$$Y = \overline{Y} + Z(s) = 250 + 1.65(75) = 373.75 \text{ women}$$

Unfortunately, a capacity of 350 is below this value, so there will not be enough space for all abused women on 95% of all nights. Obviously, the city needs at least 374 beds.

b. The area below this value is 0.3446, so the area exceeding this Z is 1 – .3446 = 0.6554. Or 65.54% of all nights the number of women seeking shelter will exceed the capacity of 220.

$$Z = \frac{220 - 250}{75} = -0.40$$

3.

a. For an individual with 13.77 years of education, his or her Z score would be

$$Z = \frac{13.77 - 13.77}{3.07} = 0.0$$

b. Since our friend's number of years of education completed is associated with the 60th percentile, we need to solve for Y. However, we must first use the logic of the normal distribution to find Z. For any normal distribution, 50% of all cases will fall above the mean. Since our friend is in the 60th percentile, we know that the area between the mean and our friend's score is 0.10. Similarly, the area beyond our friend's score is 0.40. We can now look in Appendix B column "B" for 0.10 or in column "C" for 0.40. We find that the Z associated with these values is 0.25. Now, we can solve for Y:

$$Y = \overline{Y} + Z(s) = 13.77 + 0.25(3.07) = 14.5375 = 14.54$$

c. Since we already know that the proportion between our number of years of education (13.77) and our friend's number of years of education (14.54) is 0.10, we can multiply N (1,500) by this proportion. Thus, 150 people have between 13.77 and 14.54 years of education.

4.

a. The Z score for a score of 172 is

$$\frac{172 - 140}{40} = 0.80$$

The area between the mean and Z is 0.2881. Percentile rank = 0.50 + 0.2881 = 0.7881; 100(0.7881) = 78.81st percentile.

b. The Z score for a score of 200 is

$$\frac{200 - 140}{40} = 1.50$$

c. The Z score for a score of 100 is

$$\frac{100 - 140}{40} = -1.00$$

The Z score for a score of 160 is

$$Z = \frac{160 - 140}{40} = 0.5$$

The area between a Z of −1 and the mean is 0.3413. The area between a Z of 0.5 and the mean is 0.1915, so the total area between the scores is

$$\text{Area} = 0.3413 + 0.1915 = 0.5328$$

Therefore, 53.28% of scores fall between 100 and 160.

d. A score of 190 corresponds to a Z score of

$$\frac{190 - 140}{40} = 1.25$$

Area beyond the Z is 0.1056, so about .1056 of respondents should score above 190. We are unable to calculate the exact number because the sample size was not reported in the problem.

e. A cutoff score below which 67% of the test scores fall is equivalent to a score which is below 33% of the test scores. This corresponds to a Z score of about 0.44. This translates into a test score of

$$Y = \overline{Y} + Z(s) = 140 + 0.44(40) = 157.6$$

5.

a. Among working-class respondents:

The Z score for a value of 12 is

$$Z = \frac{12 - 13.01}{2.91} = -0.35$$

The Z score for a value of 16 is

$$Z = \frac{16 - 13.01}{2.91} = 1.03$$

You'll find the area between the Z scores and the mean under Column B. The total area between the scores is .1368 + .3485 = .4853. The proportion of working-class respondents with 12 to 16 years of education is 0.4853.

Among upper-class respondents:

The Z score for a value of 12 is

$$Z = \frac{12 - 15.44}{2.83} = -1.22$$

The Z score for a value of 16 is

$$Z = \frac{16 - 15.44}{2.83} = 0.20$$

The area between a Z of −1.22 and the mean is 0.3888. The area between a Z of 0.20 is 0.0793, so the total area between the scores is 0.3888 + 0.0793 = 0.4681. The proportion of upper-class respondents with 12 to 16 years of education is 0.4681.

A higher proportion of working-class respondents have 12 to 16 years of education than upper-class respondents.

b. For working-class respondents:

As previously calculated, the Z score for a value of 16 is 1.03. The area between a Z of 1.03 and the tail of the distribution (Column C) is 0.1515. So the probability of a working-class respondent having more than 16 years of education is 15.15%.

For middle-class respondents:

The Z score for a value of 16 is

$$Z = \frac{16 - 14.99}{2.93} = .35$$

The area between a Z of .35 and the tail of the distribution (Column C) is 0.3632. So the probability of a middle-class respondent having more than 16 years of education is 36.32%.

c. For lower-class respondents:

The Z score for a value of 10 is

$$Z = \frac{10 - 12.11}{2.83} = -0.75$$

The area beyond Z of 0.75 is .2266. So the probability of a lower-class respondent having less than 10 years of education is .2266 (or 22.66%).

d. If years of education is positively skewed, then the proportion of cases with high levels of education will be less than that for a normal distribution. This means, for example, that the probabilities associated with high levels of education will be smaller.

6.

a. The area beyond the Z is about 0.1020, so 10.20% of students should score above 625.

$$Z = \frac{625 - 485}{110} = 1.27$$

b. The area between 625 ($Z = 1.27$) and the mean is 0.3980 (39.80%)

$$Z = \frac{400 - 485}{110} = -.77$$

The area between this score and the mean is 0.2794 (27.94%), so 67.74% (0.3980 + 0.2794) of all students should earn a score between 400 and 625.

c. The 20th percentile is equivalent to a Z score of –0.84. The SAT reading score equivalent is

$$Y = \overline{Y} + Z(s) = 485 + (-0.84)(110) = 392.6 = 393$$

7.

a. The Z score of 150 is 3.33.

b. The area beyond 3.33 is 0.0004. The percentage of scores above 150 is 0.04%, a very small percentage.

c. The Z score for 85 is –1.0. The percentage of scores between 85 and 150 is 84.09% (0.3413 + 0.4996 = 0.8409).

d. Scoring in the 95th percentile means that 95% of the sample scored below this level. Identifying the 95th percentile can be calculated by this formula: 100 + 1.65(15) = 124.75. The IQ score that is associated with the 95th percentile is 124.75.

8.

a. The Z score that corresponds to 300 is –1.72. The percentage of seniors who scored lower than 300 is 4.27% (.0427).

b. The Z score for 600 is 0.85; the Z score for 700 is 1.70. The percentage between 600 and 700 can be determined by subtracting area from the mean to 600 from the area from the mean to 700. The calculation is 0.4554 – 0.3023 = 0.1531 or 15.31%.

c. A Z score of 725 is 1.91. This marks the 97th percentile (0.4719 + 0.50).

9.

a. About 0.1894 of the distribution falls above the Z score, so that is the proportion of crime incidents with more than two victims.

$$Z = \frac{2 - 1.28}{0.82} = 0.88$$

b. The area between the mean and the Z score is about 0.1331, so the total area above one victim is 0.50 + 0.1331 = 0.6331, or 63.31%.

$$Z = \frac{1 - 1.28}{0.82} = -0.34$$

c. The area between the mean and the Z score is about 0.4995, so the total area below four victims is 0.50 + 0.4995 = 0.9995.

$$Z = \frac{4 - 1.28}{0.82} = 3.32$$

10.

a. The area between the value and the upper tail of the distribution is 0.1093. So the probability that someone will work more than 60 hours per week is 0.1093. This translates into approximately 98 (895 × 0.1093) respondents in the sample.

$$Z_{60} = \frac{60 - 41.47}{15.04} = 1.23$$

b. The area between 30 hours and the lower tail of the distribution is 0.2236. So the probability that someone will work less than 30 hours per week is 0.2236. This translates into approximately 200 (895 × 0.2236) respondents in the sample.

$$Z = \frac{30 - 41.47}{15.04} = -0.76$$

c. The 60th percentile corresponds to an area of 0.60; that's 0.50 + 0.10. An area of 0.10 in Column B corresponds to a Z score of approximately 0.25, which corresponds to a raw score of

$$Y = \overline{Y} + Z(s) = 41.47 + 0.25(15.04) = 45.23$$

So a raw score of 45.23 marks as the 60th percentile in this distribution.

11.

a. For a team with an APR score of 990

$$Z = \frac{990 - 981}{27.3} = 0.33$$

From Appendix B, the area beyond 0.33 is 0.3707 or about the 67th percentile. The team is not in the upper quartile.

b. The Z value which corresponds to a cutoff score with an area of about 0.25 toward the tail of the distribution is 0.67. This is translated into a cutoff score:

$$981 + 0.67(27.3) = 999.29.$$

c. The Z value is 0.67.

12.

a. For the eligibility criterion, Team A has a Z score of

$$Z = \frac{971 - 983}{33} = -0.36$$

For the retention criterion,

$$Z = \frac{958 - 976}{34.9} = -0.52$$

For the eligibility criterion, Team B has a Z score of

$$Z = \frac{987 - 983}{33} = 0.12$$

For the retention criterion,

$$Z = \frac{970 - 976}{34.9} = -0.17$$

Team B is better on eligibility and retention than Team A.

b. Team B's retention Z score was −0.17, below the mean. We refer to "Column C" and find that the proportion of teams that did worse than Team B on the retention criterion is 0.4325. This is the area between Team B's retention Z score and the tail end of the distribution.

c. Team A's Z score for eligibility is −.36. The area below the score is 0.3594. Team A's score is at the 35.94th percentile.

13. The 95th percentile corresponds to a Z score of 1.65.

Hungary

11.76 + 1.65 (2.91) = 16.56 years

Czech Republic

12.82 + 1.65 (2.29) = 16.60 years

Denmark

13.93 + 1.65 (5.83) = 23.55 years

France

14.12 + 1.65 (5.73) = 23.57 years

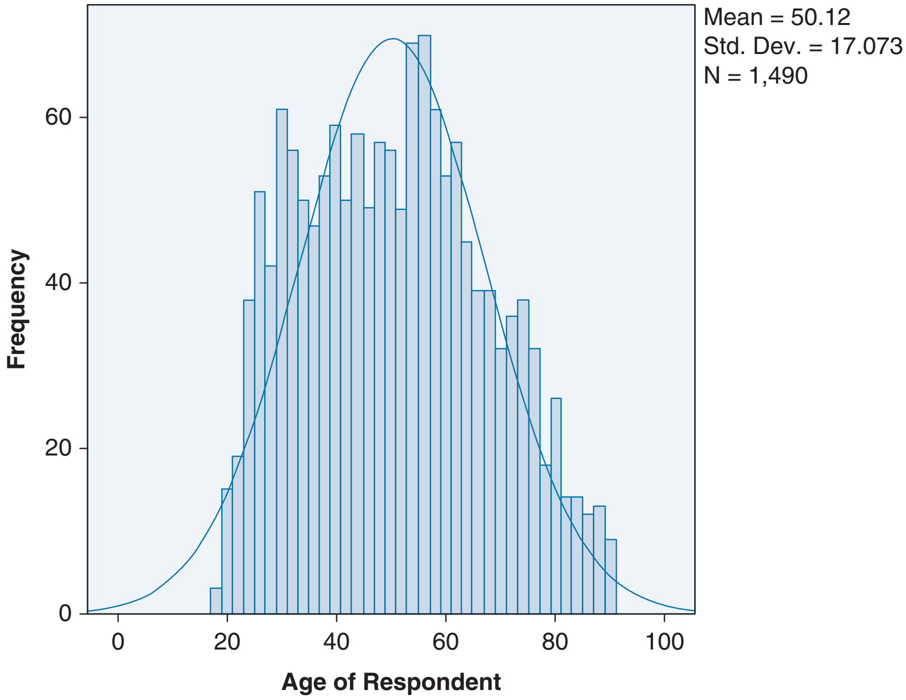
Ireland

15.15 + 1.65 (3.90) = 21.59 years

SPSS SOLUTIONS

1.

a.



Looking at the histogram for AGE, we see that the distribution for age is close to a normal distribution when compared with the normal curve. However, because of several outliers at the upper end of the distribution, the distribution itself is slightly positively skewed.

b. Below is the output for the mean and standard deviation values using the Frequencies procedure.

Statistics

age AGE OF RESPONDENT

N	Valid	1490
	Missing	10
Mean		50.12
Median		50.00
Std. Deviation		17.073

c. The Z score for the age of 25 is

$$Z = \frac{25 - 50.12}{17.07} = -1.47$$

From Appendix B, we see that the area beyond a Z score of –1.47 is 0.0708, or 7.08%. So 7.08% of the distribution falls at or below age 25.

d. Below is a portion of the frequency distribution for AGE.

age AGE OF RESPONDENT

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	18	3	.2	.2	.2
	19	7	.5	.5	.7
	20	8	.5	.5	1.2
	21	8	.5	.5	1.7
	22	11	.7	.7	2.5
	23	17	1.1	1.1	3.6
	24	21	1.4	1.4	5.0
	25	25	1.7	1.7	6.7
	26	26	1.7	1.7	8.5
	27	24	1.6	1.6	10.1
	28	18	1.2	1.2	11.3
	29	32	2.1	2.1	13.4
	30	29	1.9	1.9	15.4
	31	29	1.9	1.9	17.3
	32	27	1.8	1.8	19.1

Looking at the output, we see that 6.7% of respondents are 25 years of age or younger. This is a difference of 0.38 percentage points from our calculation.

2.

a. The easiest way to find the equivalent Z score for 18 years of education is to first switch the SPSS Data Editor Window to “Data View” (bottom left portion of the screen). Next, we locate EDUC in the columns, right click the EDUC as the column header, and select the “Sort Ascending” option. After the scores have been stratified, we scroll down the EDUC column until we find where 18 years of education begins. Finally, once we locate these values, we simply scroll over to the ZEDUC variable column and note that 1.37709 is the corresponding Z score with 18 years of education.

b. The Frequencies table shows that the cumulative percentage of respondents with 18 years or less of education is 93.7 percentage. Thus, the percentile rank is the same, 93.7.

c. The percentile rank of 93.7 is associated with a Z score of about 1.53 because we look for the Z score that has $1.00 - .937 = 0.063$ of the distribution above it. SPSS calculated a Z score of 1.37709 for 18 years, a calculation based on the assumption that the distribution of hours worked per week is normal (even though that is untrue).

d. The distributions are identical. Because SPSS chose to use a different number of intervals for each variable, the histograms don't appear identical, but they are. Transforming raw scores into Z scores changes the scale on which a variable is measured but doesn't change its distribution (mathematically, subtracting and dividing by constants preserves the order and relative magnitude of the scores).

3. For this exercise, the instructor (or students) could also select an EDUC value (other than 18 years) to calculate equivalent Z scores or to determine whether the distribution is normal for men/women. Follow the same procedures as in 2a–d.