Appendix E A Basic Math Review

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Where that this courses in statistics can involve a great deal of mathematics, you should be relieved to hear that this course will stress interpretation rather than the ability to solve complex mathematical problems. With that said, however, you will still need to know how to perform some basic mathematical operations as well as understand the meanings of certain symbols used in statistics. Following is a review of the symbols and math you will need to know to successfully complete this course.

SYMBOLS AND EXPRESSIONS USED IN STATISTICS

Statistics provides us with a set of tools for describing and analyzing *variables*. A variable is an attribute that can vary in some way. For example, a person's age is a variable because it can range from just born to more than one hundred years old. "Race" and "gender" are also variables, though with fewer categories than the variable "age." In statistics, variables you are interested in measuring are often given a symbol. For example, if we wanted to know something about the age of students in our statistics class, we would use the symbol *Y* to represent the variable "age." Now let's say for simplicity we asked only the students sitting in the first row their ages—19, 21, 23, and 32. These four ages would be scores of the *Y* variable.

Another symbol that you will frequently encounter in statistics is Σ , or uppercase sigma. Sigma is a Greek letter that stands for summation in statistics. In other words, when you see the symbol Σ , it means you should sum all of the scores. An example will make this clear. Using our sample of students' ages represented by *Y*, the use of sigma as in the expression ΣY (read as the sum of *Y*) tells us to sum all the scores of the variable *Y*. Using our example, we would find the sum of the set of scores from the variable "age" by adding the scores together:

$$19 + 21 + 23 + 32 = 95$$

So, for the variable "age," $\Sigma Y = 95$.

Sigma is also often used in expressions with an exponent, as in the expression ΣY^2 (read as the sum of squared scores). This means that we should first square all the scores of the *Y* variable and

then sum the squared products. So using the same set of scores, we would solve the expression by squaring each score first and then adding them together:

$$19^2 + 21^2 + 23^2 + 32^2 = 361 + 441 + 529 + 1,024 = 2,355$$

So, for the variable "age," $\Sigma Y^2 = 2,355$.

A similar, but slightly different, expression, which illustrates the function of parentheses, is $(\Sigma Y)^2$ (read as the sum of scores, squared). In this expression, the parentheses tell us to first sum all the scores and then square this summed total. Parentheses are often used in expressions in statistics, and they always tell us to perform the expression within the parentheses first and then the part of the problem that is outside of the parentheses. To solve this expression, we need to sum all the scores first. However, we already found that $\Sigma Y = 95$, so to solve the expression (ΣY)², we simply square this summed total,

$$95^2 = 9,025$$

So, for the variable "age," $(\Sigma Y)^2 = 9,025$.

You should also be familiar with the different symbols that denote multiplication and division. Most students are familiar with the times sign (\times); however, there are several other ways to express multiplication. For example,

$$3(4)$$
 (5)6 (4)(2) 7.8 9 * 6

all symbolize the operation of multiplication. In this text, the first three are most often used to denote multiplication. There are also several ways division can be expressed. You are probably familiar with the conventional division sign (\div) , but division can also be expressed in these other ways:

$$4/6\frac{6}{3}$$

This text uses the latter two forms to express division.

In statistics, you are likely to encounter greater than and less than signs (>, <), greater than or equal to and less than or equal to signs (\geq , \leq), and not equal to signs (\neq). It is important you understand what each sign means, though admittedly it is easy to confuse them. Use the following expressions for review. Notice that numerals and symbols are often used together:

4 > 2 means 4 is greater than 2

 $H_1 > 10$ means H_1 is greater than 10

7 < 9 means 7 is less than 9

a < b means a is less than b

 $Y \ge 10$ means that the value for Y is a value greater than or equal to 10

 $a \le b$ means that the value for *a* is less than or equal to the value for *b*

 $8 \neq 10$ means 8 does not equal 10

 $H_1 \neq H_2$ means H_1 does not equal H_2

PROPORTIONS AND PERCENTAGES

Proportions and percentages are commonly used in statistics and provide a quick way to express information about the relative frequency of some value. You should know how to find proportions and percentages.

Proportions are identified by *P*; to find a proportion, apply this formula:

$$P = \frac{f}{N}$$

where f stands for the frequency of cases in a category and N the total number of cases in all categories. So, in our sample of four students, if we wanted to know the proportion of men in the front row, there would be a total of two categories, female and male. Because there are 3 women and 1 man in our sample, our N is 4; and the number of cases in our category "male" is 1. To get the proportion, divide 1 by 4:

$$P = \frac{f}{N} \qquad P = \frac{1}{4} = .25$$

So, the proportion of men in the front row is .25. To convert this to a percentage, simply multiply the proportion by 100 or use the formula for percentaging:

$$\% = \frac{f}{N} \times 100$$
 $\% = \frac{1}{4} \times 100 = 25\%$

WORKING WITH NEGATIVES

Addition, subtraction, multiplication, division, and squared numbers are not difficult for most people; however, there are some important rules to know when working with negatives that you may need to review.

1. When adding a number that is negative, it is the same as subtracting:

$$5 + (-2) = 5 - 2 = 3$$

2. When subtracting a negative number, the sign changes:

$$8 - (-4) = 8 + 4 = 12$$

3. When multiplying or dividing a negative number, the product or quotient is always negative:

$$6 \times -4 = -24$$
 $-10 \div 5 = -2$

4. When multiplying or dividing two negative numbers, the product or quotient is always positive:

$$-3 \times -7 = 21$$
 $-12 \div -4 = 3$

5. Squaring a number that is negative always gives a positive product because it is the same as multiplying two negative numbers:

$$-5^2 = 25$$
 is the same as $-5 \times -5 = 25$

ORDER OF OPERATIONS AND COMPLEX EXPRESSIONS

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In statistics you are likely to encounter some fairly lengthy equations that require several steps to solve. To know what part of the equation to work out first, follow two basic rules. The first is called the rules of precedence. They state that you should solve all squares and square roots first, then multiplication and division, and finally, all addition and subtraction from left to right. The second rule is to solve expressions in parentheses first. If there are brackets in the equation, solve the expression within parentheses first and then the expression within the brackets. This means that parentheses and brackets can override the rules of precedence. In statistics, it is common for parentheses to control the order of calculations. These rules may seem somewhat abstract here, but a brief review of their application should make them more clear.

To solve the problem

 $4 + 6 \cdot 8 = 4 + 48 = 52$

do the multiplication first and then the addition. Not following the rules of precedence will lead to a substantially different answer:

$$4 + 6 \cdot 8 = 10 \cdot 8 = 80$$

which is incorrect.

To solve the problem

$$6-4(6)/3^2$$

first, find the square of 3,

$$6 - 4(6)/9$$

then do the multiplication and division from left to right,

$$6 - \frac{24}{9} = 6 - 2.67$$

and finally, work out the subtraction,

$$6 - 2.67 = 3.33$$

To work out the following equation, do the expressions within parentheses first:

$$(4+3) - 6(2)/(3-1)^2$$

First, solve the addition and subtraction in the parentheses,

$$(7) - 6(2)/(2)^2$$

Now that you have solved the expressions within parentheses, work out the rest of the equation based on the rules of precedence, first squaring the 2,

$$(7) - 6(2)/4$$

Then do the multiplication and division next:

$$(7) - \frac{12}{4} = (7) - 3$$

Finally, work out the subtraction to solve the equation:

$$7 - 3 = 4$$

The following equation may seem intimidating at first, but by solving it in steps and following the rules, even these complex equations should become manageable:

$$\sqrt{(8(4-2)^2)/(12/4)^2}$$

For this equation, work out the expressions within parentheses first; note that there are parentheses within parentheses. In this case, work out the inner parentheses first,

$$\sqrt{\left(8(2)^2\right)/3^2}$$

Now do the outer parentheses, making sure to follow the rules of precedence within the parentheses—square first and then multiply:

$$\sqrt{\frac{32}{3^2}}$$

Now, work out the square of 3 first and then divide:

$$\sqrt{\frac{32}{9}} = \sqrt{3.55}$$

Last, take the square root: